HYPOTHESIS TESTING – AN OVERVIEW

- We use hypothesis tests to determine if there is a statistically significant difference between two groups.
- There are two types:
    1. Comparing **sample data to population data** (the "status quo")

    Ex: The population mean SAT score is 1600, however we believe that Rutgers University has a higher than average mean SAT score. By using a representative sample of Rutgers students, we can determine if the mean SAT score of Rutgers students is greater than that of the population.

    2. Comparing **two sample means** (x-bar$_1$ and x-bar$_2$) of sample groups (n$_1$ and n$_2$)

    Ex: Professor Salaries – We want to determine whether there is a difference in the mean salaries of professors from public and private universities. We take the annual salaries of 35 (n$_1$) faculty members of private institutions and 30 (n$_2$) faculty members of public universities. Using this approach we can determine whether or not there is a difference in the mean pay for professors based on the type of institution they work for.
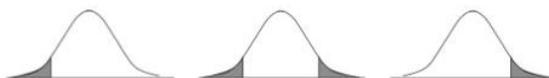
PROCEDURE

1. **Null Hypothesis H$_o$:**

    $\mu_0 = \mu$ (population mean provided in problem)
    $\mu_1 = \mu_2$ (comparing 2 means)

    Alternate Hypothesis H$_a$ (or H$_1$):

    $\mu_1 > \mu_2$ or $\mu_0 > \mu$ – One-Tailed (right tailed)
    $\mu_1 < \mu_2$ or $\mu_0 < \mu$ – One-Tailed (left tailed)
    $\mu_1 \neq \mu_2$ or $\mu_o \neq \mu$ – Two-Tailed

2. **Determine Test Type (One-Tailed or Two-Tailed)**

3. **Use the FORMULAS ON THE FRONT**
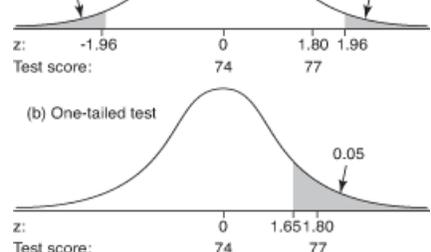
4. **Understand what you are doing:**
   - Realize the area under the curve = 1
   - This means that for an alpha level of 0.05, the area of the rejection region = 0.05
   - Therefore, in order to find the upper and/or lower critical values for a Z Test, search for 0.05 and/or 0.95 in the CENTER of the table and find its equivalent Z score (the Critical Value)
   - For a T Test it is crucial to use df = n–1 in order to find the correct Critical Value
   - Under a Standard Normal Curve, the mean = 0. This means that any Critical Value on a Z or T Table is equal in both directions (i.e. Assume alpha = 0.05, for a Right Tailed test we would look for 0.95 in the center of the table and find the Z score to be 1.64. Similarly for a Left Tailed test we would look for 0.05 in the center and find -1.64 as the Critical. This also applies to T Test Critical Values)
   - Remember to divide the alpha by 2 for Two Tailed test and look for the new value ($\alpha/2$) in the center of the table to find the Critical Value

One-Tail Test (left tail)	Two-Tail Test	One-Tail Test (right tail)
$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$
$H_1 : \mu < \mu_0$	$H_1 : \mu \neq \mu_0$	$H_1 : \mu > \mu_0$

Ex. If alpha = 0.05 for a Z test the critical value is 1.64

Ex. If alpha = 0.05 and the df = 5 for a T test the critical value is 2.015

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706

t Table

cum. prob one-tail	$t_{.50}$ 0.50	$t_{.75}$ 0.25	$t_{.80}$ 0.20	$t_{.85}$ 0.15	$t_{.90}$ 0.10	$t_{.95}$ 0.05	$t_{.975}$ 0.025	$t_{.99}$ 0.01	$t_{.995}$ 0.005	$t_{.999}$ 0.001	$t_{.9995}$ 0.0005
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221

Two tests at the same probability level (5%)

Two-tailed test

0.025 0.025

z: -1.96 0 1.80 1.96
Test score: 74 77

(b) One-tailed test

0.05

z: 0 1.651.80
Test score: 74 77

5. **Compare the Testing Number and Critical Value**

   ✓ **For a Two Tailed Test** – If the testing number is greater than the upper critical value or less than the lower critical value we can say the testing number is in the rejection region and can REJECT the H$_o$

   ✓ **For a One Tailed Test** – If it is right tailed and the Testing number is greater than the critical value we can REJECT the H$_o$. If it is left tailed and the Testing number is less than the critical value we can REJECT the H$_o$.

Works Cited

Weiss, Neil A. Introductory Statistics. 9th ed. Boston, MA: Pearson Education Inc, 2012. Print.

STATISTICS FORMULA SHEET & RULES FOR HYPOTHESIS TESTING

Sample Mean: $\overline{X}$

Sample Std Dev: s

Population Mean: μ

Population Std Dev: σ

n = Sample Size

df = Degrees of Freedom

H_o = Null Hypothesis (The "status quo" what is already accepted)

H_a = Alternate Hypothesis (What is trying to be proven as correct or different from the status quo)

Hypothesis Testing Procedure

- Z – Test when n > 30
- T – Test when n < 30
- For Z and T tests:

1. Write your null and alternate hypothesis
2. Determine whether the test is One Tailed or Two Tailed.

   **Is it One Tailed is it Right or Left Tailed? Is it a One Sample or Two Sample Test?

   **If it is Two Tailed remember to divide the alpha value by 2 because the rejection region is now split in half on both tails

3. Calculate the Z or T value (Testing Number) using the equation
4. Find the Critical Value from the Z or T table based on the alpha level (i.e. 0.05, 0.10)
5. Compare the Testing Number to the Critical Value

In Step 5, realize the significance of whether the test is one tailed or two tailed AND whether it is Left or Right Tailed

- ✓ **For a Two Tailed Test** – If the testing number is greater than the upper critical value or less than the lower critical value we can say the testing number is in the rejection region and can reject the H_o
- ✓ **For a One Tailed Test** – If it is right tailed and the Testing number is greater than the critical value we can REJECT the H_o. If it is left tailed and the Testing number is less than the critical value we can REJECT the H_o.

One Sample Z Test

One Sample T Test
$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}},$$

Two Sample Z Test
$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}},$$

One Proportion Z Test
$$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Two Proportion Z Test
$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

One Sample Z Confidence Interval (CI)
$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \quad \text{to} \quad \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}},$$

One Sample T Confidence Interval (CI)
$$\bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \quad \text{to} \quad \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}},$$

One Proportion Z Confidence Interval
$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Two Proportion Z Confidence Interval
$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Two Sample T Test & Confidence Interval (CI)

Pooled Two Sample T Test	Unpooled Two Sample T Test
Assumed Equal Std Devs	**Assumed Unequal Std Devs**

Pooled:
$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}.$$
$$\bar{x}_1 - \bar{x}_2$$
$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \cdot s_p\sqrt{(1/n_1) + (1/n_2)}.$$

Unpooled:
$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}$$
$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \cdot \sqrt{(s_1^2/n_1) + (s_2^2/n_2)}.$$